**GEOMETRIC AND ARITHMETIC PROGRESSION IN SOLVING APPLIED PROBLEMS**

**Abstract**: This article discusses problem solving using arithmetic and geometric progressions with practical applications.

**Keywords**: progression, arithmetic and geometric progression, progression term, sum of progression terms

Progression is a sequence of quantities, each subsequent one of which is in some common dependence on the previous one. Many processes that occur in our lives, which can be described by numerical characteristics, can be studied using progressions. In mathematics, two types of progressions are considered: arithmetic and geometric. An arithmetic progression is a sequence where each term, starting from the second one, is equal to the previous term plus the same number. A geometric progression is a sequence of non-zero numbers, where each term, starting from the second one, is equal to the previous term multiplied by the same number.

Students get acquainted with the concepts of these types of progressions in the 9th grade. When studying these topics, they not only analyze the definitions of types of progressions, but also learn to identify which type of progression is represented by a sequence of quantities. In addition, each progression is described by formulas that allow finding a term of a given progression, as well as the sum of the terms of this progression. Problems solved using progressions are included in the assessment materials of the OGE and the Unified State Exam.

In this article, we will look at examples of solving applied problems using geometric and arithmetic progressions. Such problems create interdisciplinary connections between the studied objects and disciplines, develop creative thinking, and lay the foundation for financial literacy skills.

Let's consider a practical problem that can be offered to students for solving. "Builders are laying a tunnel 50 meters long, increasing the laying rate by the same distance each day. It is known that on the first day, builders laid 4 meters of the tunnel. Determine how many meters of the tunnel the builders laid on the last day if all the work was completed in 10 days. The builders started their work on August 8th, a Wednesday. Calculate the completion date of the builders' work, considering that they have a day off on Sunday. Calculate the total earnings of the entire team of builders on the last day of work, if the team earned 20,000 rubles on the first day." Calculate how much each worker in the team will receive after deducting income tax at 13%, if the team consists of 5 people?"

After reading the condition of this problem, we notice that the work rate increases by the same number each day. Therefore, we can solve this problem using the concept of an arithmetic progression. The first term of this progression is 4, as 4 meters of the tunnel were laid on the first day, the sum of all terms in the arithmetic progression is 50, as a total of 50 meters need to be laid, and the number of terms in the progression is 10, as the workers worked for 10 days. To find the last term of this progression, we will use the formula for calculating the sum of an arithmetic progression, which looks like the sum $s\_{n}=\frac{a\_{1}+a\_{n}}{2}⋅n$. Substitute the known values into this formula, we get $50=\frac{4+a\_{10}}{2}⋅10$. Then we get $50=\left(4+a\_{10}\right)⋅5 $and reduce 50 and 5 then we obtain the equation $10=4+a\_{10} $hence, on the last day, 6 metres of tunnel was laid. Now let's find the completion date of the tunnel construction work by the team. They started working on August 8th, a Wednesday - the first day of work. According to the schedule, they are supposed to work for 10 days, with Sunday being a day off. So they will have 4 working days in the first week and 6 working days in the second week. Therefore, they will finish their work on Saturday, August 19th. Now let's calculate the workers' earnings on the last day of tunnel construction. According to the problem statement, on the first day, the workers laid 4 meters of the tunnel and earned 20,000 rubles. Then we calculate the cost of laying one meter of the tunnel by dividing 20,000 by 4, which equals 5,000 rubles. Since on the last day they laid 6 meters of the tunnel, the payment for the work done will be 5,000 $∙$ 6 $=$ 30,000 rubles. Now let's answer the last question of our problem. For the construction of the entire tunnel, the entire team will receive the following amount: 5,000 $∙ $50 $=$ 250,000 rubles. There are 5 people in the team, so each worker's earnings before income tax deduction will be 50,000 rubles. Our income tax deduction is 13% of the received amount. Let's find this amount in rubles. To do this, we divide 50,000 rubles by 100 to find 1%, and then multiply by 13%. We get the following result: 50,000:100 $∙$3=6,500 rubles. Now subtract this amount from 50,000 rubles, and the final earnings after tax deduction will be 43,500 rubles.

Now let's consider the application of a geometric progression in solving real-world problems. "6th grade student Zhenya Volkov, returning from a walk, forgot to wash his hands and sat down to eat. During his meal, 40 dysentery bacteria entered his intestines. Every 20 minutes, the number of bacteria triples. How many bacteria will be in Zhenya's intestines in 2 hours? Will he get dysentery during this period if he does not take medication, given that more than 12,000 bacteria are needed for the disease to develop?"

Carefully reading the problem statement, students should notice that the number of bacteria triples, meaning it increases threefold. Thus, we can conclude that this problem can be solved using a geometric progression. According to the problem, we need to find the number of bacteria that will form during this time period. To do this, we will use the formula for the sum of a geometric progression, which looks like this:$ S\_{n}=\frac{b\_{1}\left(q^{n}-1\right)}{q-1}$ . In this formula, $b\_{1}$ is the first term of the geometric progression, which in our problem equals 40, q is the progression ratio, which is 3 as the number of bacteria triples. Now let's find n, the number of times the bacteria tripling process occurred. Since the problem mentions two hours and tripling occurs every 20 minutes, we have n = 6. Now let's calculate the total number of bacteria: $s\_{6}=\frac{40⋅\left(3^{6}-1\right)}{3-1}=\frac{40⋅\left(729-1\right)}{2}=20⋅728=14560$ . As we can see, the number of bacteria exceeds the permissible limit of 12,000, meaning Zhenya will get sick if he doesn't take the necessary medication. Therefore, one should remember one of the important hygiene rules, which is to wash hands when coming in from outside!

**References**

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